Unit - 1 Automata Fundamentals

Finite Automata (FA):

1) Deterministic FA
2) non - deterministic FA

◯ ⇒ start state

◎ ⇒ Final State

⟶ ⇒ Transition function

a,....,z } → Input Symbals
0,....,9

Ex: I/P 'ab'

Start $\rightarrow$ (1) $\xrightarrow{a}$ (2) $\xrightarrow{b}$ ((3))

I/P 'aab'

Start $\rightarrow$ (1) $\xrightarrow{a}$ (2) $\xrightarrow{b}$ ((3)) ; loop a on (2)

1) Constract Finite Automata for the set of strings {a,b} Start state is 'a' Ending state 'b'

'ab', 'aab', 'aabb', 'abab', 'aaaaab', 'abbbbb', 'aababb'

ab : Start $\rightarrow$ (1) $\xrightarrow{a}$ (2) $\xrightarrow{b}$ ((3))

aab : Start $\rightarrow$ (1) $\xrightarrow{a}$ (2) $\xrightarrow{b}$ ((3)) ; loop a on (2)

aabb : Start $\rightarrow$ (1) $\xrightarrow{a}$ (2) $\xrightarrow{b}$ ((3)) ; loop a/b on (2)

abab : Start $\rightarrow$ (1) $\xrightarrow{a}$ (2) $\xrightarrow{b}$ ((3)) ; loop a/b on (2)

aaaaab }
abbbbb } : Start $\rightarrow$ (1) $\xrightarrow{a}$ (2) $\xrightarrow{b}$ ((3)) ; loop a/b on (2)

DFA
NFA

2) Constract finite Automata for the set of String (0,1) start with '00' Ending with '11'

'0011', '00011', '000011', '00111', '00111', '000011111'

0011 : Start $\rightarrow$ (a) $\xrightarrow{0}$ (b) $\xrightarrow{0}$ (c) $\xrightarrow{1}$ (d) $\xrightarrow{1}$ ((e))

00011 : $\xrightarrow{S}$ (a) $\xrightarrow{0}$ (b) $\xrightarrow{0}$ (c) $\xrightarrow{1}$ (d) $\xrightarrow{1}$ ((e)) ; loop 0 on (c)

000011 : $\xrightarrow{S}$ (a) $\xrightarrow{0}$ (b) $\xrightarrow{0}$ (c) $\xrightarrow{1}$ (d) $\xrightarrow{1}$ ((e)) ; loop 0 on (c)

00111 : $\xrightarrow{S}$ (a) $\xrightarrow{0}$ (b) $\xrightarrow{0}$ (c) $\xrightarrow{1}$ (d) $\xrightarrow{1}$ ((e)) ; loop 0/1 on (c)

00111 }
000011111 } : $\xrightarrow{S}$ (a) $\xrightarrow{0}$ (b) $\xrightarrow{0}$ (c) $\xrightarrow{1}$ (d) $\xrightarrow{1}$ ((e)) ; loop 0/1 on (c)

Theory of Computations

It is a discrib the basic idea & model underlying Computing.

Computation is the Process of execution of an Algorithm such that it involves taking some I/P & Performing required operations on it to Produce an O/P

Process

I/P → $\boxed{\text{Function (F)}}$ → O/P

Computation function.

Application:

- Complier design
- ~~Robo stick~~, Robotics
- Artificial Intelligence
- Knowledge Engineering

Finite Automata:

The finite Automata is a mathematical model of a system with discrete I/P & O/P

It Consists of finite set of states & a set of transition from one state to another state that depends on I/P Symbols chosen from alphabet Σ.

from definition of FA:

The finite Automata can be defined as five pupils people's.

$$M = \{Q, \Sigma, \delta, q_0, F\}$$

Where,

- Q - finite set of states
- Σ - finite set of I/P symbols
- δ - transistion function
- q_0 - starting state / Initial state
- F - Final state

types of finite Automata:

i) deterministic FA (DFA)

ii) non-determinstic FA (NFA)

i) DFA:

DFA is defined as "If there is a single Path for a single I/P from current state to next state."

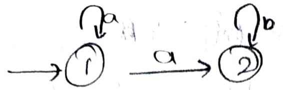
Eg: $\rightarrow (1) \xrightarrow{a} (2)$

$$M = \{Q, \Sigma, \delta, q_0, F\}$$

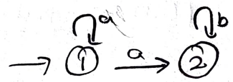
ii) NFA:

NFA is defined as "If there is a multiple Path for a specified I/P from current state to next state.

Eg:



$M = \{Q, \Sigma, \delta, q_0, F\}$

DFA	NFA
If there is a Single Path for a Single I/p from Current state to next State.	If there is a multiple Path for a Specified I/p from Current State to next state.
It doesn't have "ε" Transaction.	It does have "ε" Transaction.
Eg:  $M = \{Q, \Sigma, \delta, q_0, F\}$	Eg:  $M = \{Q, \Sigma, \delta, q_0, F\}$

Transaction Diagram:

The DFA Can be represented as directed graph namely transaction Diagram.

Transaction function:

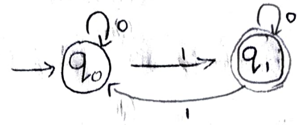
$\delta(q_0, 0) = q_0$

$\delta(q_0, 1) = q_1$

$\delta(q_1, 0) = q_1$

$\delta(q_1, 1) = q_0$

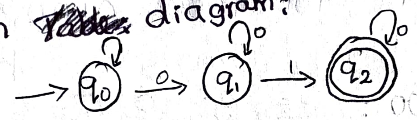
Transaction Diagram:



Transaction Table:

Transition table is the tabular representation of the transition function of DFA whose rows denotes the States & Column denotes the I/P symbols.

Transition diagram:



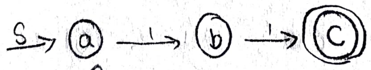
Transition table:

	0	1
$\rightarrow q_0$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_1\}$	$\{q_2\}$
$* q_2$	$\{q_2\}$	ϕ

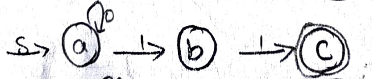
1) Constract DFA that accept I/P string 0's & 1's are [0,1] that End with '11'

11, 011, 0011, 0111, 1011, 00011, 011111, 10011

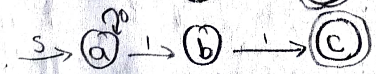
11 :



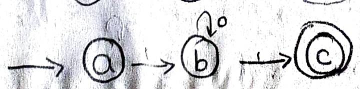
011 :



0011 :



1011



0111 : $\xrightarrow{s} (a)^{0} \xrightarrow{1} (b) \xrightarrow{1} ((c))^{1}$

00011 : $\xrightarrow{s} (a)^{0} \xrightarrow{1} (b) \xrightarrow{1} ((c))$

01111 : $\xrightarrow{s} (a)^{0} \xrightarrow{1} (b) \xrightarrow{1} ((c))^{1}$

10011 : $\xrightarrow{s} (a)^{0} \xrightarrow{1} (b)^{0} \xrightarrow{1} ((c))^{1}$

2) Constract DFA that accept I/p string 0's & 1's are (0,1) the end with '00'

00, 100, 1000, 0100, 10011

00 : $\xrightarrow{s} (a) \xrightarrow{0} (b) \xrightarrow{0} ((c))$

100 : $\xrightarrow{s} (a)^{1} \xrightarrow{0} (b) \xrightarrow{0} ((c))$

1000 : $\xrightarrow{s} (a)^{1} \xrightarrow{0} (b)^{0} \xrightarrow{0} ((c))$

0100 : $\xrightarrow{s} (a) \xrightarrow{0} (b)^{1} \xrightarrow{0} ((c))^{0}$

10000 : $\rightarrow (a)^{1} \xrightarrow{0} (b) \xrightarrow{0} ((c))^{0}$

10100 $\rightarrow (a) \xrightarrow{0} (b) \xrightarrow{0} ((c))^{0}$ 1

0010100

3. Constract DFA for a set of I/P String {0,1} with 3 Consecutive zero in End (000)

000, 1000, 101000, 10000, 101000, 110000

000 : $\rightarrow (a) \xrightarrow{0} (b) \xrightarrow{0} (c) \xrightarrow{0} ((d))$

11000 : $\rightarrow (a)^{1} \xrightarrow{0} (b) \xrightarrow{0} (c) \xrightarrow{0} ((d))$

101000 : $\rightarrow (a)^{1} \xrightarrow{0} (b)^{1} \xrightarrow{0} (c) \xrightarrow{0} ((d))^{0}$

1010000 : $\rightarrow (a)^{1} \xrightarrow{0} (b)^{1} \xrightarrow{0} (c) \xrightarrow{0} ((d))^{0}$

10101000 : $\rightarrow (a)^{1} \xrightarrow{0} (b)^{1} \xrightarrow{0} (c)^{1} \xrightarrow{0} ((d))^{0}$

110000 : $\rightarrow (a)^{1} \xrightarrow{0} (b) \xrightarrow{0} (c) \xrightarrow{0} ((d))^{0}$

4. Construct DFA for the set of String {0,1} that as even no. of 0 & 1.

0011, 1010, 0101, 000011, 110000, 0101010, 10101010

0011 :

1010 :

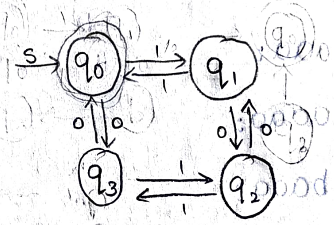
0101 :

000011 :

110000 :

01010101 :

10101010 :



19 non-deterministic finite Automata :

Constract NFA for the set of all strings {0,1} The ending with 11.

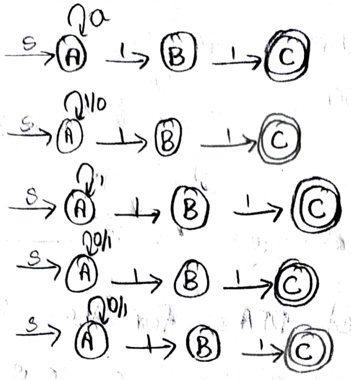
11, 011, 0111, 0011, 1011, 00011, 01111, 010011

11 : $\rightarrow (A) \xrightarrow{1} (B) \xrightarrow{1} ((C))$

011 : $\rightarrow (A)^{0} \xrightarrow{1} (B) \xrightarrow{1} ((C))$

0111 : $\rightarrow (A)^{0/1} \xrightarrow{1} (B) \xrightarrow{1} ((C))$

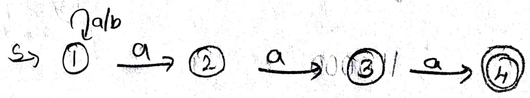
0011 :
1011 :
00011 :
01111 :
010011 :



2 Construct NFA for the set of all strings {a,b} at end with 3 consitative A's

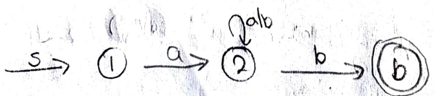
aaa, aaaa, baaa, abaaa, bbaaa, bbaaa

aaa:
aaaa:
baaa:
abaaa:
baaaa:
bbaaa:



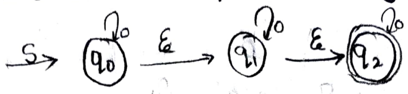
3. Construct NFA for the set of string a's & b's starting with a & Ending with b.

ab, aab, abb, aabb, aaab, aaabb



ϵ - closure:

1) Find ϵ - closure for the following FA:

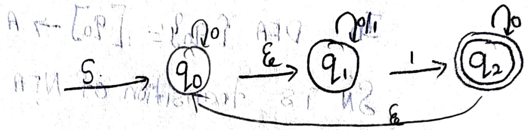


ϵ - closure $\{q_0\} \to \{q_0, q_1, q_2\}$

ϵ - closure $\{q_1\} \to \{q_0, q_2\}$

ϵ - closure $\{q_2\} \to \{q_2\}$

2) Find a ϵ - closure for the following NFA:



ϵ - closure $\{q_0\}$ - $\{q_0, q_1\}$

ϵ - closure $\{q_1\}$ - $\{q_1\}$

ϵ - closure $\{q_2\}$ - $\{q_2, q_0, q_1\}$

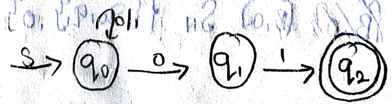
big qu 1) Problem on NFA to DFA:

methods: 1) NFA to DFA

2) NFA with ϵ to NFA without ϵ

3) NFA with ϵ to DFA

Convert the following NFA to DFA:



Soln:

Transition table for the above ~~NFA~~ is

	0	1
$\rightarrow q_0$	$\{q_0,q_1\}$	$\{q_0\}$
q_1	ϕ	$\{q_2\}$
$* q_2$	ϕ	ϕ

Here Strating state is $\{q_0\}$ in NFA.

In DFA $\{q_0\} = [q_0] \rightarrow A$

δ_N is transition of NFA

δ_D is transition of DFA

The I/P symbols are $\{0,1\}$

Step 1:

$(A,0)$ $\delta_N\{\{q_0\},0\} = \{q_0,q_1\}$ $(A,0)\ \delta_N\{\{q_0\},0\} = \{q_0,q_1\}$

So $\delta_D\{\{q_0\},0\} = \{q_0,q_1\} \rightarrow B$

$(A,1)$ $\delta_N\{\{q_0\},1\} = \{q_0\}$

So $\delta_D\{\{q_0\},1\} = \{q_0\} \rightarrow A$

Step 2:

$(B,0)$ $\delta_N\{\{q_0,q_1\},0\} = \{\{q_0,0\} \cup \{q_1,0\}\}$

$= \{q_0,q_1\} \cup \phi$

$= \{q_0,q_1\}$

So $\delta_D\{\{q_0,q_1\},0\} = \{q_0,q_1\} \rightarrow B$

$(B,1)$ $\delta_N\{\{q_0,q_1\},1\} = \{\{q_0,1\} \cup \{q_1,1\}\}$

$= \{q_0\} \cup \{q_2\}$

$= \{q_0,q_2\}$

So $\delta_D\{\{q_0,q_1\},1\} = \{q_0,q_2\} \rightarrow C$

Step 3:

$(C,0)$ $\delta_N\{\{q_0,q_2\},0\} = \{\{q_0,0\} \cup \{q_2,0\}\}$

$= \{q_0,q_1\} \cup \phi = q_0,q_1$

So $\delta_D\{\{q_0,q_2\},0\} = q_0,q_1 \rightarrow B$

$(C,1)$ $\delta_N\{\{q_0,q_2\},1\} = \{\{q_0,1\} \cup \{q_2,1\}\}$

$= \{q_0\} \cup \phi$

$= \{q_0\}$

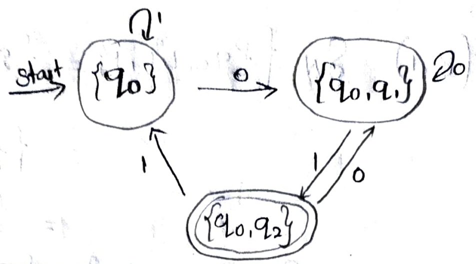
So $\delta_D\{\{q_0,q_2\},1\} = \{q_0\} \rightarrow A$

transsistion table for DFA:

	0	1
A	B	A
B	B	C
C	B	A

	0	1
$\rightarrow \{q_0\}$	$\{q_0,q_1\}$	$\{q_0\}$
$\{q_0,q_1\}$	$\{q_0,q_1\}$	$\{q_0,q_2\}$
$* \{q_0,q_2\}$	$\{q_0,q_1\}$	$\{q_0\}$

Transition diagram for DFA :

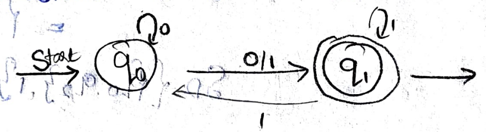


2. Construct DFA from the following NFA

	0	1
$\rightarrow q_0$	$\{q_0,q_1\}$	$\{q_1\}$
$* q_1$	ϕ	$\{q_0,q_1\}$

Soln:

transition diagram for the above NFA is



Here : starting state is $\{q_0\}$ in NFA

In DFA $\{q_0\} = [q_0] \rightarrow A$

δ_N is transition of NFA

δ_D is " of DFA

Step 1:

$(A,0)$ $\delta_N\{\{q_0\},0\} = \{q_0,q_1\}$

So $\delta_D\{\{q_0\},0\} = \{q_0,q_1\} \rightarrow B$

$(A,1)$ $\delta_N\{\{q_0\},1\} = \{q_1\}$

So $\delta_D\{\{q_0\},1\} = \{q_1\} \rightarrow C$

Step 2:

$(B,0)$ $\delta_N\{\{q_0,q_1\},0\} = \{\{q_0,0\} \cup \{q_1,0\}\}$

$= \{q_0,q_1\} \cup \phi$

$= \{q_0,q_1\}$

So $\{\{q_0,q_1\},0\} = \{q_0,q_1\} \rightarrow B$

$(B,1)$ $\delta_N\{q_0,q_1\},0\} = \{\{q_0,1\} \cup \{q_1,1\}\}$

$= \{q_1\} \cup \{q_0,q_1\}$

So $\{q_0,q_1\},0\} = \{q_0,q_1\} \rightarrow B$

Step 3:

$(C,0)$ $\delta_N\{\{q_1\},0\} = \{q_1,0\} = \phi$

$\delta_D\{\{q_1\},0\} = \phi$

$(C,1)$ $\delta_N\{\{q_1\},1\} = \{q_1,1\}$

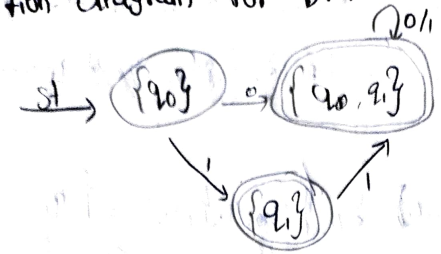
$\delta_D\{\{q_1\},1\} = \{q_0,q_1\} \rightarrow B$

transition table for DFA

	0	1
$\rightarrow \{q_0\}$	$\{q_0,q_1\}$	$\{q_1\}$
$\{q_0,q_1\}$	$\{q_0,q_1\}$	$\{q_0,q_1\}$
$* \{q_1\}$	ϕ	$\{q_0,q_1\}$

	0	1
A	B	C
B	B	B
C	ϕ	B

Transition diagram for DFA:

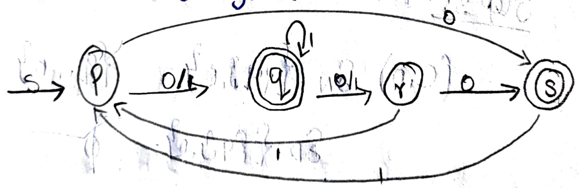


3. Construct DFA to the following NFA:

	0	1
$\rightarrow p$	$\{q,s\}$	$\{q\}$
$*q$	$\{r\}$	$\{q,r\}$
r	$\{s\}$	$\{p\}$
$*s$	ϕ	$\{p\}$

Soln:

transition diagram for the above NFA is



here Starting state is $\{p\}$ in NFA

In DFA $\{p\} = [p] \rightarrow A$

δ_N is transition of NFA

δ_D is transition of DFA

Step 1:

$(A,0)$ $\delta_N\{\{p\},0\} \Rightarrow \{p,0\}$

$= \{q,s\}$

$\delta_D\{\{p\},0\} = \{q,s\} \rightarrow B$

$(A,1)$ $\delta_N\{\{p\},1\} = \{p,1\}$

$\delta_D\{\{p\},1\} = \{q\} \rightarrow C$

Step 2:

$(B,0)$ $\delta_N\{\{q,s\},0\} = \{\{q,0\} \cup \{s,0\}\}$

$= \{r\} \cup \phi$

$\delta_D\{\{q,s\},0\} = \{r\} \rightarrow D$

$(B,1)$ $\delta_N\{\{q,s\},1\} = \{\{q,1\} \cup \{s,1\}\}$

$= \{q,r\} \cup \{p\}$

$\delta_D\{\{q,s\},1\} = \{p,q,r\} \rightarrow E$

Step 3:

$(C,0)$ $\delta_N\{\{q\},0\} = \{q,0\}$

$= r$

$\delta_D\{\{q\},0\} = \{r\} \rightarrow D$

$(C,1)$ $\delta_N\{\{q\},1\} = \{q,1\} = \{q,r\}$

$\delta_D\{\{q\},1\} = \{q,r\} \rightarrow F$

Step 4:

$(D,0)$ $\delta_N\{\{r\},0\} = \{r,0\} = \{s\}$

$\delta_D\{\{r\},0\} = \{s\} \rightarrow G$

$(D,1)$ $\delta_N\{\{r\},1\} = \{r,1\} = \{p\}$

$\delta_D\{\{r\},1\} = \{p\} \rightarrow A$

Step 5:

$(E,0)$ $S_N\{\{p,q,r\},0\} = \{\{p,0\} \cup \{q,0\} \cup \{r,0\}\}$

$= \{q,s\} \cup \{r\} \cup \{s\} = \{q,r,s\}$

$S_D\{\{p,q,r\},0\} = \{q,r,s\} \rightarrow H$

$(E,1)$ $S_N\{\{p,q,r\},1\} = \{\{p,1\} \cup \{q,1\} \cup \{r,1\}\}$

$= \{q\} \cup \{q,r\} \cup \{p\} = \{p,q,r\}$

$S_D\{\{p,q,r\},1\} = \{p,q,r\} \rightarrow E$

Step 6:

$(F,0)$ $S_N\{\{q,r\},0\} = \{\{q,0\} \cup \{r,0\}\}$

$= \{r\} \cup \{s\} = \{r,s\}$

$S_D\{\{q,r\},1\} = \{r,s\} \rightarrow I$

$(F,1)$ $S_N\{\{q,r\},1\} = \{\{q,1\} \cup \{r,1\}\}$

$= \{q,r\} \cup \{p\} = \{p,q,r\}$

$S_D\{\{q,r\},1\} = \{p,q,r\} \rightarrow E$

Step 7:

$(G,0)$ $S_N\{\{s\},0\} = \{s,0\} = \phi$

$S_D\{\{s\},0\} = \phi$

$(G,1)$ $S_N\{\{s\},1\} = \{s,1\} = \{p\}$

$S_D\{\{s\},1\} = \{p\} \rightarrow A$

Step 8:

$(H,0)$ $S_N\{\{q,r,s\},0\} = \{\{q,0\} \cup \{r,0\} \cup \{s,0\}\}$

$= \{r\} \cup \{s\} \cup \phi = \{r,s\}$

$S_D\{\{q,r,s\},0\} = \{r,s\} \rightarrow I$

$(H,1)$ $S_N\{\{q,r,s\},1\} = \{\{q,1\} \cup \{r,1\} \cup \{s,1\}\}$

$= \{q,r\} \cup \{p\} \cup \{p\} = \{p,q,r\}$

$S_D\{\{q,r,s\},1\} = \{p,q,r\} \rightarrow E$

Step 9:

$(I,0)$ $S_N\{\{r,s\},0\} = \{\{r,0\} \cup \{s,0\}\}$

$= \{s\} \cup \{\phi\} = \{s\}$

$S_D\{\{r,s\},0\} = \{s\} \rightarrow G$

$(I,1)$ $S_N\{\{r,s\},1\} = \{\{r,1\} \cup \{s,1\}\}$

$= \{\{p\} \cup \{p\}\} = \{p\}$

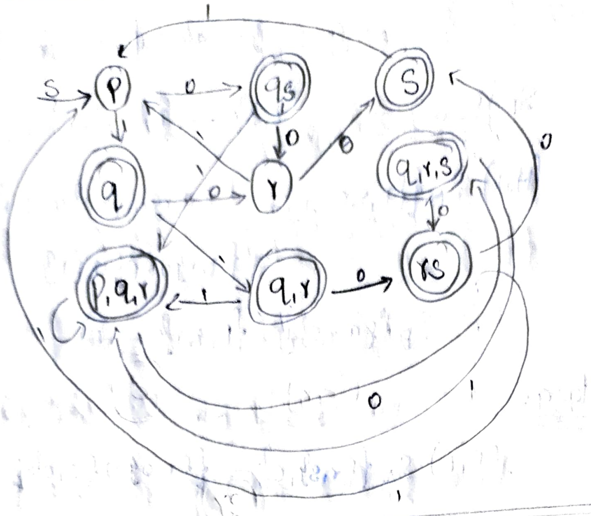
$S_D\{\{r,s\},1\} = \{p\} \rightarrow A$

transition table fo DFA:

	0	1
$\rightarrow \{p\}$	$\{q,s\}$	$\{q\}$
$* \{q,s\}$	$\{r\}$	$\{p,q,r\}$
$* \{q\}$	$\{r\}$	$\{q,r\}$
$\{r\}$	$\{s\}$	$\{p\}$
$* \{p,q,r\}$	$\{q,r,s\}$	$\{p,q,r\}$
$* \{q,r\}$	$\{r,s\}$	$\{p,q,r\}$
$* \{s\}$	ϕ	$\{p\}$
$* \{q,r,s\}$	$\{r,s\}$	$\{p,q,r\}$
$* \{r,s\}$	$\{s\}$	$\{p\}$

	0	1
A	B	C
B	D	E
C	D	F
D	G	A
E	H	E
F	I	E
G	ϕ	A
H	I	E
I	G	A

transition diagram for DFA:

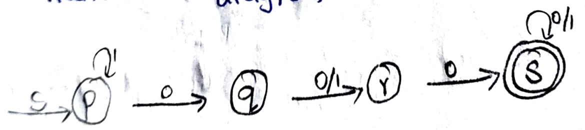


Convert DFA the following NFA:

	0	1
$\rightarrow p$	$\{p,q\}$	$\{p\}$
q	$\{r\}$	$\{r\}$
r	$\{s\}$	ϕ
$*s$	$\{s\}$	$\{s\}$

Soln:

transition diagram for the above NFA



here starting state is $\{p\}$ in NFA

In DFA $\{p\} = [p] \rightarrow A$

δ_N is transition of NFA

δ_D is " of DFA

Step 1:

$(A,0)$ $\delta_N \{\{p\},0\} = \{p,0\}$

$= \{p,q\}$

$\delta_D \{\{p\},0\} = \{p,q\} \rightarrow B$

$(A,1)$ $\delta_N \{\{p\},1\} = \{p,1\} = \{p\}$

$\delta_D \{ \quad = \{p\} \rightarrow A$

Step 2:

$(B,0)$ $\delta_N \{\{p,q\},0\} = \{\{p,0\} \cup \{q,0\}\}$

$= \{p,q\} \cup \{r\} = \{p,q,r\}$

$\delta_D \{\{p,q\},0\} = \{p,q,r\} \rightarrow C$

$(B,1)$ $\delta_N \{\{p,q\},1\} = \{\{p,1\} \cup \{q,1\}\}$

$= \{p\} \cup \{r\} = \{p,r\}$

$\delta_D \{\{p,q\},1\} = \{p,r\} \rightarrow D$

Step 3:

$(C,0)$ $\delta_N \{\{p,q,r\},0\} = \{\{p,0\} \cup \{q,0\} \cup \{r,0\}\}$

$= \{p,q\} \cup \{r\} \cup \{s\}$

$= \{p,q,r,s\}$

$\delta_D \{\{p,q,r\},0\} = \{p,q,r,s\} \rightarrow E$

$(C,1)$ $\delta_N \{\{p,q,r\},1\} = \{\{p,1\} \cup \{q,1\} \cup \{r,1\}\}$

$= \{p\} \cup \{r\} \cup \{\phi\}$

$= \{p,r\}$

$\delta_D \{\{p,q,r\},1\} = \{p,r\} \rightarrow D$

Step 4:

$(D,0)$ $\delta_N \{\{p,r\},0\} = \{\{p,0\} \cup \{r,0\}\}$

$= \{\{p,q\} \cup \{s\}\} = \{p,q,s\}$

$S_D\{\{p,r\},0\} = \{p,q,s\} \rightarrow$ (F)

$(D,1)$ $\{\{p,r\},1\} = \{\{p,1\}, \{r,1\}\}$

$= \{p\} \cup \{\phi\} = \{p\}$

$S_D\{\{p,r\},1\} = \{p\} \rightarrow$ (A)

Step 5:

$(E,0)$ $S_N\{\{p,q,r,s\},0\} = \{\{p,0\}, \{q,0\}, \{r,0\}, \{s,0\}\}$

$= \{\{p,q\} \cup \{r\} \cup \{s\} \cup \{s\}\}$

$= \{p,q,r,s\}$

$S_D\{\{p,q,r,s\},0\} = \{p,q,r,s\} \rightarrow$ (E)

$(E,1)$ $S_N\{\{p,q,r,s\},1\} = \{\{p,1\}, \{q,1\}, \{r,1\}, \{s,1\}\}$

$= \{p\} \cup \{r\} \cup \{\phi\} \cup \{s\} = \{p,r,s\}$

$S_D\{\{p,q,r,s\},1\} = \{p,r,s\} \rightarrow$ (G)

Step 6:

$(F,0)$ $S_N\{\{p,q,s\},0\} = \{\{p,0\}, \{q,0\}, \{s,0\}\}$

$= \{\{p,q\} \cup \{r\} \cup \{s\}\} = \{p,q,r,s\}$

$S_D\{\{p,q,s\},0\} = \{p,q,r,s\} \rightarrow$ (E)

$(F,1)$ $S_N\{\{p,q,s\},1\} = \{\{p,1\}, \{q,1\}, \{s,1\}\}$

$= \{p\} \cup \{r\} \cup \{s\} = \{p,r,s\}$

$S_D\{\{p,q,s\},1\} = \{p,r,s\} \rightarrow$ (G)

Step 7:

$(G,0)$ $S_N\{\{p,r,s\},0\} = \{\{p,0\}, \{r,0\}, \{s,0\}\}$

$= \{p,q\} \cup \{s\}, \{s\}$

$= \{p,q,s\}$

$S_D\{\{p,r,s\},0\} = \{p,q,s\} \rightarrow$ (F)

$(G,1)$ $S_N\{\{p,r,s\},1\} = \{\{p,1\}, \{r,1\}, \{s,1\}\}$

$= \{p\} \cup \{\phi\} \cup \{s\} = \{p,s\}$

$S_D\{\{p,r,s\},1\} = \{p,s\} \rightarrow$ (H)

Step 8:

$(H,0)$ $S_N\{\{p,s\},0\} = \{\{p,0\}, \{s,0\}\}$

$= \{p,q\} \cup \{s\} = \{p,q,s\}$

$S_D\{\{p,s\},0\} = \{p,q,s\} \rightarrow$ (F)

$(H,1)$ $S_N\{\{p,s\},1\} = \{\{p,1\}, \{s,1\}\}$

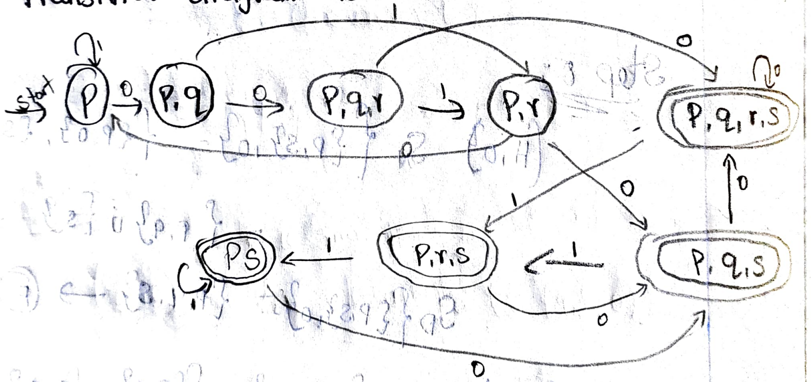
$= \{p\} \cup \{s\} = \{p,s\}$

$S_D\{\{p,s\},1\} = \{p,s\} \rightarrow$ (H) transition table for DFA:

	0	1
→{p}	{p,q}	{p}
{p,q}	{p,q,r}	{p,r}
{p,q,r}	{p,q,r,s}	{p,r}
{p,r}	{p,q,s}	{p}
*{p,q,r,s}	{p,q,r,s}	{p,r,s}
*{p,q,s}	{p,q,r,s}	{p,r,s}
*{p,r,s}	{p,q,s}	{p,s}
*{p,s}	{p,q,s}	{p,s}

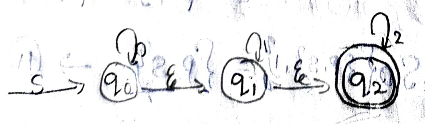
	0	1
A	B	A
B	C	D
C	G	D
D	F	A
E	E	G
F	E	G
G	F	H
H	F	H

transition diagram for DFA:



Problem on NFA with ϵ to NFA without ϵ

1.



Soln:

transition table for NFA with ϵ

	0	1	2	ϵ
$\rightarrow q_0$	$\{q_0\}$	ϕ	ϕ	$\{q_1\}$
q_1	ϕ	$\{q_1\}$	ϕ	$\{q_2\}$
$* q_2$	ϕ	ϕ	$\{q_2\}$	ϕ

Step 1:

ϵ-closure $\{q_0\} = \{q_0, q_1, q_2\}$

ϵ-closure $\{q_1\} = \{q_1, q_2\}$

ϵ-closure $\{q_2\} = \{q_2\}$

Step 2:

Processing state $\{q_0\}$

meaning for ϵ-closure $\{q_0\}$

$\delta'(q_0, 0) = \epsilon\text{-closure}\{\delta(\delta'\{q_0\}, 0)\}$

~~$= \epsilon$-closure $\{\delta(\{q_0, q_1, q_2\}, 0)\}$~~

$\delta(\delta'\{q_0\}, 0)$

$= \epsilon\text{-closure}\{\delta(\{q_0, q_1, q_2\}, 0)\}$

$q_0, 0$ — 0, 1, 2

$= \epsilon\text{-closure}\{\delta(q_0, 0) \cup (q_1, 0) \cup (q_2, 0)\}$

$= \epsilon\text{-closure}\{\{q_0\} \cup \phi \cup \phi\}$

$= \epsilon\text{-closure}\{q_0\}$

$= \{q_0, q_1, q_2\}$

$\delta'(q_0, 1) = \epsilon\text{-closure}\{\delta(\delta'\{q_0\}, 1)\}$

$= \epsilon\text{-closure}\{\delta(\{q_0, q_1, q_2\}, 1)\}$

$= \epsilon\text{-closure}\{\delta(q_0, 1) \cup (q_1, 1) \cup (q_2, 1)\}$

$= \epsilon\text{-closure}\{\phi \cup \{q_2\} \cup \phi\}$

$= \epsilon\text{-closure}\{q_2\}$

$= \{q, q_2\}$

$(q_0, 2) = \epsilon\text{-closure}\{\delta(\delta'\{q_0\}, 2)\}$

$= \epsilon\text{-closure}\{\delta\{q_0, q_1, q_2\}, 2\}$

$= \epsilon\text{-closure}\{\delta(q_0, 2) \cup (q_1, 2) \cup (q_2, 2)\}$

$= \epsilon\text{-closure}\{\phi \cup \phi \cup \{q_2\}\}$

$= \epsilon\text{-closure}\{q_2\}$

$= \{q_2\}$

Step 3:

Processing state $\{q_1\}$

$(q_1, 0) = \epsilon\text{-closure}\{\delta(\delta'\{q_1\}, 0)\}$

$= \epsilon\text{-closure}\{\delta\{q_1, q_2\}, 0\}$

$= \epsilon\text{-closure}\{\delta\{q_1, 0\} \cup \{q_2, 0\}\}$

$= \epsilon\text{-closure}\{\phi \cup \phi\}$

$= \phi$

$(q_1, 1) = \epsilon\text{-closure}\{\delta(\delta'\{q_1\}, 1)\}$

$= \epsilon\text{-closure}\{\delta(\delta\{q_1, q_2\}, 1)\}$

$= \epsilon\text{-closure}\{\delta\{q_1, 1\} \cup \{q_2, 1\}\}$

$= \epsilon\text{-closure}\{q_1\} \cup \phi$

$= \epsilon\text{-closure}\{q_1\}$

$= \{q_1, q_2\}$

$(q_1, 2) = \epsilon\text{-closure}\{\delta(\delta'\{q_1\}, 2)\}$

$= \epsilon\text{-closure}\{\delta(\delta\{q_1, q_2\}, 2)\}$

$= \epsilon\text{-closure}\{\delta(\delta\{q_1, 2\} \cup \{q_2, 2\}\}$

$= \epsilon\text{-closure}\{\phi \cup \{q_2\}\}$

$= \epsilon\text{-closure}\{q_2\}$

$= \{q_2\}$

Step 4:

Processing state $\{q_2\}$

$(q_2, 0) = \epsilon\text{-closure}\{\delta\{\delta'\{q_2\}, 0)\}$

$= \epsilon\text{-closure}\{\delta\{q_2\}, 0\}$

$= \epsilon\text{-closure}\{\delta\{q_2, 0\}\}$

$= \epsilon\text{-closure}\{\phi\} = \phi$

$(q_2, 1) = \varepsilon\text{-closure}\{\delta(\delta'\{q_2\}, 1)\}$

$= \varepsilon\text{-closure}\{\delta\{q_2\}, 1\}$

$= \varepsilon\text{-closure}\{\delta\{q_2, 1\}\}$

$= \varepsilon\text{-closure}\{\phi\}$

$= \phi$

$(q_2, 2) = \varepsilon\text{-closure}\{\delta(\delta'\{q_2\}, 2\}$

$= \varepsilon\text{-closure}\{\delta\{q_2, 2\}\}$

$= \varepsilon\text{-closure}\{\delta\{q_2, 2\}\}$

$= \varepsilon\text{-closure}\{q_2\}$

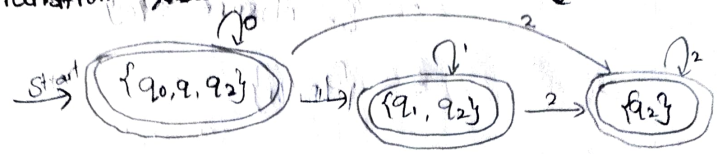
$= \{q_2\}$

transition table for NFA without ε-closure:

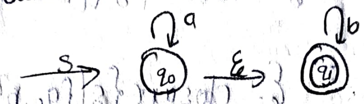
	0	1	2
$\rightarrow q_0$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
$* q_2$	ϕ	ϕ	$\{q_2\}$

	0	1	2
$\rightarrow * \{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
$* \{q_1, q_2\}$		$\{q_1, q_2\}$	$\{q_2\}$
$* \{q_2\}$			$\{q_2\}$

transition diagram for NFA without ε-closure:



2. Consider NFA with ε closure to NFA without ε-closure



Soln:

transition table for NFA with ε

	a	b	ε
$\rightarrow q_0$	$\{q_0\}$	ϕ	$\{q_1\}$
$* q_1$	ϕ	$\{q_1\}$	ϕ

Step 1:

$\varepsilon\text{-closure}\{q_0\} = \{q_0, q_1\}$

$\varepsilon\text{-closure}\{q_1\} = \{q_1\}$

Step 2:

Processing state $\{q_0\}$

$(q_0, a) = \varepsilon\text{-closure}\{\delta(\delta'\{q_0\}, a)\}$

$= \varepsilon\text{-closure}\{\delta(\{q_0, q_1\}, a)\}$

$= \varepsilon\text{-closure}\{\delta\{q_0, a\} \cup \{q_1, a\}\}$

$= \varepsilon\text{-closure}\{q_0\} \cup \{\phi$

$= \epsilon\text{-closure}\{q_0\}$

$= \{q_0, q_1\}$

$(q_0, b) = \epsilon\text{-closure}\{\delta(\delta'\{q_0\}, b)\}$

$= \epsilon\text{-closure}\{\delta(\{q_0, q_1\}, b)\}$

$= \epsilon\text{-closure}\{\delta(\{q_0, b\} \cup \{q_1, b\})\}$

$= \epsilon\text{-closure}\{\phi\} \cup \{q_1\}$

$= \epsilon\text{-closure}\{q_1\}$

$= \{q_1\}$

Step 3:

Processing state $\{q_1\}$

$(q_1, a) = \epsilon\text{-closure}\{\delta(\delta'\{q_1\}, a)\}$

$= \epsilon\text{-closure}\{\delta(\{q_1, a\})\}$

$= \epsilon\text{-closure}\{\phi\}$

$= \phi$

$(q_1, b) = \epsilon\text{-closure}\{\delta(\delta'\{q_1\}, b)\}$

$= \epsilon\text{-closure}\{\delta(\{q_1, b\})\}$

$= \epsilon\text{-closure}\{q_1\}$

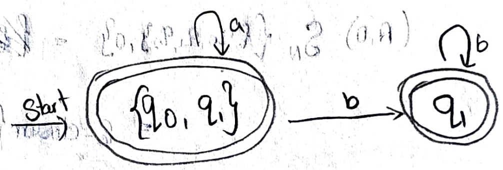
$= \{q_1\}$

transition table for NFA without ε-closure

	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$* q_1$	ϕ	$\{q_1\}$

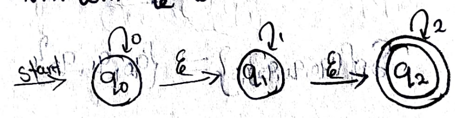
	a	b
$\rightarrow * \{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_1\}$
$* q_1$	ϕ	$\{q_1\}$

transition ~~state~~ diagram for NFA without ε



Problem for Converting NFA with ε to DFA:

1. Convert NFA with ε to DFA



Soln:

Transition table for NFA with ε

	0	1	2	ε
$\rightarrow q_0$	$\{q_0\}$	ϕ	ϕ	$\{q_1\}$
q_1	ϕ	$\{q_1\}$	ϕ	$\{q_2\}$
$* q_2$	ϕ	ϕ	$\{q_2\}$	ϕ

Step 1:

ϵ-closure $\{q_0\} = \{q_0, q_1, q_2\}$

ϵ-closure $\{q_1\} = \{q_1, q_2\}$

ϵ closure $\{q_2\} = \{q_2\}$

Assume that the initial state in the above NFA is $\{q_0\} = \epsilon$-closure $\{q_0\}$

$= \{q_0, q_1, q_2\} \rightarrow$ A

Step 2:

(A,0) $\delta_N \{\{q_0, q_1, q_2\}, 0\} = \epsilon$-closure $\{\{q_0, q_1, q_2\}, 0\}$

$= \epsilon$-closure $\{(q_0, 0) \cup (q_1, 0) \cup (q_2, 0)\}$

$= \epsilon$-closure $\{\{q_0\} \cup \phi \cup \phi\}$

$= \epsilon$-closure $\{q_0\} = \{q_0, q_1, q_2\}$

$\delta_D\{\{q_0, q_1, q_2\}, 0\} = \{q_0, q_1, q_2\} \rightarrow$ A

(A,1) $\delta_N \{\{q_0, q_1, q_2\}, 1\} = \epsilon$-closure $\{\{q_0, q_1, q_2\}, 1\}$

$= \epsilon$-closure $\{(q_0, 1) \cup (q_1, 1) \cup (q_2, 1)\}$

$= \epsilon$-closure $\{\phi \cup \{q_1\} \cup \phi\}$

$= \epsilon$-closure $\{q_1\} = \{q_1, q_2\}$

$\delta_D\{\{q_0, q_1, q_2\}, 1\} = \{q_1, q_2\} \rightarrow$ B

(A,2) $\delta_N \{\{q_0, q_1, q_2\}, 2\} = \epsilon$-closure $\{\{q_0, q_1, q_2\}, 2\}$

$= \epsilon$-closure $\{(q_0, 2) \cup (q_1, 2) \cup (q_2, 2)\}$

$= \epsilon$-closure $\{\phi \cup \phi \{q_2\}\}$

$= \epsilon$-closure $\{q_2\} = \{q_2\}$

$\delta_D\{\{q_0, q_1, q_2\}, 2\} = \{q_2\} \rightarrow$ C

Step 3:

(B,0) $\delta_N \{\{q_1, q_2\}, 0\} = \epsilon$ closure $\{\{q_1, q_2\}, 0\}$

$= \epsilon$-closure $\{(q_1, 0) \cup (q_2, 0)\}$

$= \epsilon$-closure $\{\phi \cup \phi\}$

$=$ ~~ϵ-closure~~ $\{\phi\}$

$\delta_D\{\{q_1, q_2\}, 0\} = \phi$

(B,1) $\delta_N \{\{q_1, q_2\}, 1\} = \epsilon$-closure $\{\{q_1, q_2\}, 1\}$

$= \epsilon$-closure $\{\{q_1, 1\} \cup \{q_2, 1\}\}$

$= \epsilon$-closure $\{\{q_1\} \cup \phi\}$

$= \epsilon$-closure $\{q_1\} = \{q_1, q_2\}$

$\delta_D\{\{q_1, q_2\}, 1\} = \{q_1, q_2\} \rightarrow$ B

(B,2) $\delta_N \{\{q_1, q_2\}, 2\} = \epsilon$-closure $\{\{q_1, q_2\}, 2\}$

$= \epsilon$-closure $\{\{q_1, 2\} \cup \{q_2, 2\}$

$= \epsilon\text{-closure}\{\{\phi\} \cup \{q_2\}\}$

$= \epsilon\text{-closure}\{q_2\} = \{q_2\}$

$\delta_D\{\{q_1, q_2\}, 2\} = \{q_2\} \rightarrow C$

Step 4:

$(C, 0)$ $\delta_N\{\{q_2\}, 0\} = \epsilon\text{-closure}\{\{q_2, 0\}\}$

$= \epsilon\text{-closure}\{\phi\} = \phi$

$\delta_D\{\{q_2\}, 0\} = \phi$

$(C, 1)$ $\delta_N\{\{q_2\}, 1\} = \epsilon\text{-closure}\{\{q_2, 1\}\}$

$= \epsilon\text{-closure}\{\phi\} = \phi$

$\delta_D\{\{q_2\}, 1\} = \phi$

$(C, 2)$ $\delta_N\{\{q_2\}, 2\} = \epsilon\text{-closure}\{\{q_2, 2\}\}$

$= \epsilon\text{-closure}\{q_2\} = \{q_2\}$

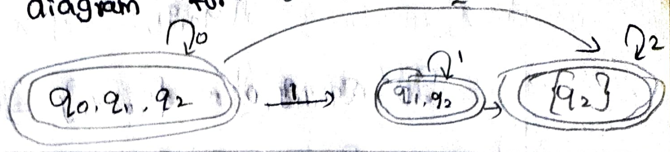
$\delta_D\{\{q_2\}, 2\} = \{q_2\} \rightarrow C$

transition table for DFA:

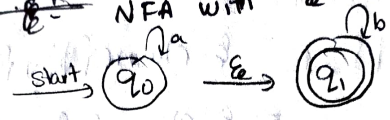
	0	1	2
$\rightarrow * \{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
$* \{q_1, q_2\}$	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
$* \{q_2\}$	ϕ	ϕ	$\{q_2\}$

	0	1	2
A	A	B	C
B	ϕ	B	C
C	ϕ	ϕ	C

transition diagram for DFA:



~~NFA with ϵ~~ NFA with ϵ to DFA:



Soln:

transition table for NFA with ϵ

	a	b	ϵ
$\rightarrow q_0$	$\{q_0\}$	ϕ	$\{q_1\}$
$* q_1$	ϕ	q_1	ϕ

Step 1:

$\epsilon\text{-closure}\{q_0\} = \{q_0, q_1\} \rightarrow A$

$\epsilon\text{-closure}\{q_1\} = \{q_1\}$

Assume that the initial state in the above NFA is

$\boxed{\{q_0\} = \epsilon\text{-closure}\{q_0\}}$

$= \{q_0, q_1\} \rightarrow A$

Step 2:

~~$(A, a)\ \delta_N\{q_0, ...$~~

(A, a) $\delta_N\{\{q_0, q_1\}, a\} = \epsilon\text{-closure}\{\{q_0, q_1\}, a\}$

$= \epsilon\text{-closure}\{\{q_0, a\} \cup \{q_1, a\}\}$

$= \epsilon\text{-closure}\{\{q_0\} \cup \{\phi\}\}$

$= \epsilon\text{-closure}\{q_0\}$

$= \{q_0, q_1\}$

$\delta_D\{\{q_0,q_1\}, a\} = \{q_0, q_1\} \rightarrow (A)$

(A,b) $\delta_N\{\{q_0,q_1\}, b\} =$ ε-closure $\{\{q_0, b\} \cup \{q_1, b\}\}$

$= \varepsilon$-closure$\{\phi\} \cup \{q_1\}$

$= \varepsilon$-Closure $\{q_1\} = \{q_1\}$

$\delta_D\{\{q_0,q_1\}, b\} = \{q_1\} \rightarrow (B)$

step 3:

(B,a) $\delta_N\{\{q_1\}, a\} = \varepsilon$-closure $\{\{q_1, a\}\}$

$= \varepsilon$-closure $\{\phi\} = \phi$

$\delta_D\{\{q_1\}, a\} = \phi$

(B,b) $\delta_N\{\{q_1\}, b\} = \varepsilon$-closure$\{\{q_1, b\}\}$

$= \varepsilon$-closure$\{q_1\} = \{q_1\}$

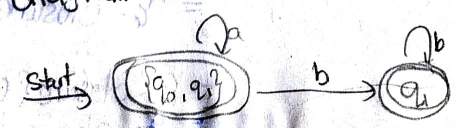
$\delta_D\{\{q_1\}, b\} = \{q_1\} \rightarrow (B)$

transition table for DFA:

	a	b
$\rightarrow * \{q_0,q_1\}$	$\{q_0,q_1\}$	$\{q_1\}$
$* \{q_1\}$	ϕ	$\{q_1\}$

	a	b
A	A	B
B	ϕ	B

transition diagram for DFA:



Equivalents of NFA & DFA

theorem:

If a regular language L is accepted by a non-deterministic finite automata then there exists a deterministic finite automata that accepted L.

(or)

A regular language L is accepted by sum DFA if & only if L is accepted by sum NFA

(or)

As, Every DFA is an NFA, the classes of languages accepted by NFA includes the classes of language accepted by DFA.

(or)

DFA can simulate NFA.

To Proof: $L(M_N) = L(M_D)$

where $M_N \rightarrow$ is the finite Automata for NFA.

$M_D \rightarrow$ is the finite Automata for DFA

Prove:

let, $M_N = \{Q_N, \Sigma, \delta_N, q_0, F_N\}$

we a NFA accepting regular language L:

let, us Constract $M_D = \{Q_D, \Sigma, \delta_D, q_0, F_D\}$

The state of MD are all the Supset of State MN

Set of states of $\boxed{DFA = 2^Q}$ Set of states of NFA

The element ~~of~~ (or) states of DFA Q_D, will be denoted by $[q_0, q_1, q_2, \ldots q_i]$ where $[q_0, q_1, q_2, \ldots q_i\}$ in NFA

In ~~DFA~~ NFA (or) QN

Thus, we define $\delta_D\{(q_0, q_1, \ldots q_i), a\} = \{P_0, P_1 \ldots P_n\}$

If and only if $\delta_N\{(q_0, q_1, \ldots q_i), a\} = \{P_0, P_1, \ldots P_n\}$

&

It is easy to show by index of the I/P String x, that $\delta_D\{\{q_0\}, x\} = \{q_0, q_1, \ldots q_n\}$

If an only if $\delta_N\{\{q_0\}, x\} = \{q_0, q_1, q_2 \ldots q_n\}$

In duction:

let x, greather or equal to each "$x \geq 1$" that Proof for string length ≥ 1.

The length of the String "x" is $|x|$. Be the string = "xa" such that

$\delta_D\{q_0, xa\} = \delta_D\{\delta_D(q_0, x), a\} \rightarrow ①$

if and only if $\delta_D\{q_0, x\} = \{q_0, q_1, \ldots q_n\} \rightarrow ②$

$\delta_N\{q_0, x\} = \{q_0, q_1, \ldots q_n\}$

Applying ② in ①

$\delta_D\{q_0, xa\} = \delta_D\{\{q_0, q_1, \ldots q_n\}, a\}$

$\delta_D\{q_0, xa\} = \{r_1, r_2 \ldots r_n\}$ (or) $\{P_1, P_2 \ldots P_n\}$

if and only if

$\delta_N\{q_0, xa\} = \{r_1, r_2 \ldots r_n\}$ $\{P_1, P_2 \ldots P_n\}$

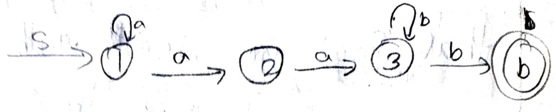
So, Completely Prove $\delta_D\{q_0, x)$ is in "FD" Exactly when, $\delta_N\{q_0, xa)$ is in "FN."

$L(M_D) = L(M_N)$ is Proof

1) define Finite Automata	6) Construct DFA for the set of String $\{a, b\}$ starting state aa, End state b
2) define DFA	7) NFA with for set of String (0,1) starting state 00, End state 11
3) " NFA	
4) diff b/w DFA & NFA	
5 Application of TOC	

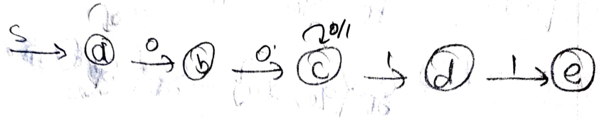
DFA: 'aa' 'b'

aab, aaab, aabb, aaabb, aaaab



NFA:

00, 11, 00011, 00111, 000111, 000111

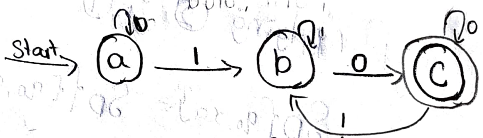


2mark:

Construct DFA

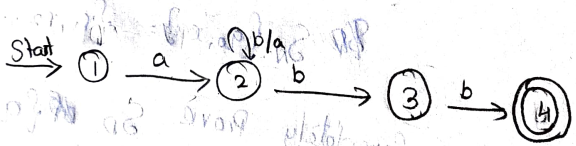
1. for the set of string {0,1} ending state is (10)

Ex: 110, 010, 0110, 1010, 0010, 00010, 01110, 101000010



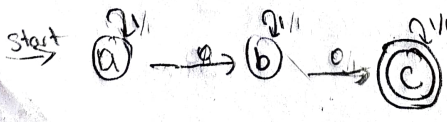
2. Construct NFA for the set of string {a,b} the Starting state 'a' & Ending state 'bb'.

abb, abbb, aabb, aabbb, aababb



3. Construct NFA for the set of string {0,1} string Contains minimum 2 0's

00, 000, 100, 0000, 1100,



Introduction to formal Proof:

Deductive to Proof:

If $x \geq 4$ the $2^x \geq x^2$

If x is the Sum of the square of four Positive integers, then $2^x \geq x^2$

Statement:

1) $x = a^2 + b^2 + c^2 + d^2$
2) $a \geq 1, b \geq 1, c \geq 1, d \geq 1$
3) $a^2 \geq 1, b^2 \geq 1, c^2 \geq 1, d^2 \geq 1$
4) $x \geq 4$
5) $2^x = x^2$

- If then,
    1) x is real no if then $|x| = |x|$
    2) If and only if x is a Real no if and only if x is integer.

Additional forms of Proof:

1) Proofs about set
2) Proofs about Contradiction
3) Proofs about Counter Example
4) Proof about Contrapositive

Proofs about set:

Ex: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof about Contrapositive:

1) x & y is true
2) x is true y is false
3) x is false y is true
4) x & y is false

Proof about Contradiction

Ex: $A \cup (B \cap C)$ "If then" $(A \cup B) \cap (A \cup C)$

$A \cup (B \cap C)$ "all and only" $(A \cup B) \cap (A \cup C)$

Proof about Counter Example:

integer 2 is Prime no but 2 is even

~~intex~~
Indective Proof:

Proof: indective method

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

Proof:

$n=1$
$n=k$
$n=k+1$

$n=1 \Rightarrow 1 = \frac{1(1+1)}{2} = \frac{2}{2} = 1$

$n=2 \Rightarrow 2 = \frac{2(2+1)}{2} = 3$

$n=3 \Rightarrow 3 = \frac{3(3+1)}{2} = \frac{3(4)}{2} = 6$

$n=4 \Rightarrow 4 = \frac{4(4+1)}{2} = 2(5) = 10$

If n is true the k is also true

If k is true then k+1 is also true.

$$1+2+3+\dots k = \frac{k(k+1)}{2}$$

$$1+2+3+\dots+k+(k+1) = \frac{k(k+1)((k+1)+1)}{2}$$

$$\frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2}$$

$$\frac{k(k+1)+2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

$2k+2$ = RHS

$+3k+2$ = RHS

$(k+1)(k+2)$ = RHS