

21-7-19

Unit - 3

Context Free Grammar (CFG)

The Context free grammar is a finite set of variables each of which represent a language.

The language represent by variable are described recursively in terms of each other primitive symbols called terminal.

The rules relating the variable are called Production.

non terminal: A, B, \dots, Z

terminal: $a, \dots, z, +, -, *, /, \dots, \epsilon$

1) find the Context free Grammar for the following language: $L = \{0^n \mid n \geq 1\}$

Soln:

$$L = \{0^n \mid n \geq 1\}$$

$$\begin{cases} S \rightarrow B \\ B \rightarrow 0 \end{cases}$$

$$\begin{cases} S \rightarrow A \\ A \rightarrow 0 \\ S \rightarrow A, A \\ A \rightarrow 0/A/\epsilon \\ S \rightarrow A, A, A \\ A \rightarrow 0 \end{cases}$$

$$\begin{cases} S \rightarrow A \\ A \rightarrow 0 \end{cases}$$

$$\begin{cases} S \rightarrow B \\ B \rightarrow 0 \end{cases}$$

2. find the CFG for the following language $L = \{0^n \mid n \geq 1\}$

Soln:

$$L = \{0^n \mid n \geq 1\}$$

$$\begin{cases} S \rightarrow A \\ A \rightarrow 0/A/\epsilon \end{cases}$$

$$\begin{aligned} L &= 0^1 \\ &= 0^2 = 0 \cdot 0 \\ &= 0^3 = 0 \cdot 0 \cdot 0 \end{aligned}$$

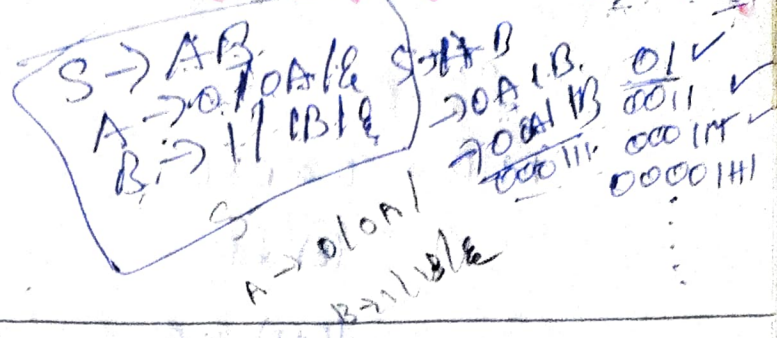
$$\begin{cases} S \rightarrow A \\ A \rightarrow 0/A/\epsilon \end{cases}$$

$$\begin{cases} S \rightarrow 0AA \\ A \rightarrow 0AA/\epsilon \end{cases}$$

3. Find the CFG for the following language $L = \{0^n 1^n \mid n \geq 1\}$

Soln:

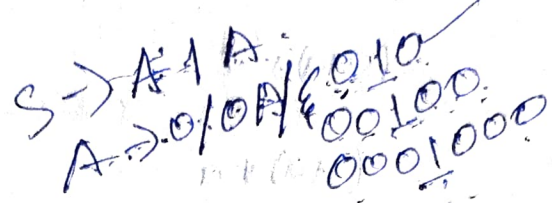
$$\begin{aligned}
 S &\rightarrow SB \\
 A &\rightarrow 0/0A/\epsilon \\
 B &\rightarrow 1/1B/\epsilon
 \end{aligned}$$



4. $L = \{0^n 10^n \mid n \geq 1\}$

Soln:

$$\begin{aligned}
 S &\rightarrow A1A \\
 A &\rightarrow 0/0A/\epsilon
 \end{aligned}$$



Parse tree (or) derivation tree

- i) left most derivation
- ii) Right " " "

i) find left most & Right most derivation for the following equation $E \rightarrow E+E / E * E / (E) / a$ by the production 'a + a * a'

Soln:

left most derivation

$$\begin{aligned}
 (1) E &\rightarrow E+E \\
 (1-1) E &\rightarrow E+E * E \\
 (1-1) E &\rightarrow a+E * E \\
 (1-1) E &\rightarrow a+a * E \\
 (1-1) E &\rightarrow a+a * a
 \end{aligned}$$

Right most derivation

$$\begin{aligned}
 (1) E &\rightarrow E+E \\
 (1-1) E &\rightarrow E + E * E \\
 (1-1) E &\rightarrow E + E * a \\
 (1-1) E &\rightarrow E + a * a \\
 (1-1) E &\rightarrow a + a * a
 \end{aligned}$$

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2) $(a+a) * a$

LMD $\Rightarrow E \rightarrow E * E$

$(E) * E$

$(E+E) * E \rightarrow (a+E) * E$

$(a+a) * E$

$(a+a) * a$

RMD $\Rightarrow E * E$

$(E) * E$

$(E+E) * E$

~~$(a+a) * E$~~ $\rightarrow (E+E) * a$
 $\rightarrow (E+a) * a$

$(a+a) * a$

3) $E \rightarrow E + E \mid E * E \mid E - E \mid (E) \mid a \quad (a * a) + (a - a)$

LED:

$E \rightarrow E + E$

$\Rightarrow (E) + E$

$\Rightarrow (E * E) + E$

$\Rightarrow (E * E) + (E)$

$\Rightarrow (E * E) + (E - E)$

$\Rightarrow (E * E) + (E - a)$

$\Rightarrow (a * a) + (E - E)$

$\Rightarrow (a * a) + (a - E)$

$E \Rightarrow (a * a) + (a - a)$

RMD:

$E \rightarrow E + E$

~~$\Rightarrow (E) + (E)$~~ $\Rightarrow E + (E)$

$\Rightarrow E + (E - E)$

$\Rightarrow (E) + (E - E)$

$\Rightarrow (E * E) + (E - E)$

$\Rightarrow (E * E) + (E - a)$

$\Rightarrow (E * E) + (a - a)$

$\Rightarrow (E * a) + (a - a)$

$E \Rightarrow (a * a) + (a - a)$

$$(4) E \rightarrow E+E / E \times E / \overset{E-E}{\wedge}(E) / a \quad (a + (a \times a))$$

LED:

$$\begin{aligned} E &\rightarrow (E) \\ &\rightarrow (E+E) \\ &\rightarrow (E+(E)) \\ &\rightarrow (E+(E \times E)) \\ &\rightarrow (a+(E \times E)) \\ &\rightarrow (a+(a \times E)) \\ E &\rightarrow (a+(a \times a)) \end{aligned}$$

RED:

$$\begin{aligned} E &\rightarrow (E) \\ &\rightarrow (E+E) \\ &\rightarrow (E+(E)) \\ &\rightarrow (E+(E \times E)) \\ &\rightarrow (E+(E \times a)) \\ &\rightarrow (E+(a \times a)) \\ E &\rightarrow (a+(a \times a)) \end{aligned}$$

1. derive the string $b \times (b + a_{11})$ using LMD derivation & RMD derivation from the production $E \rightarrow I / E+E / E \times E / (E)$

$$I \rightarrow a/b / I_a / I_b / I_0 / I_1$$

Soln:

LED:

$$\begin{aligned} E &\rightarrow E \times E \\ &\rightarrow E \times (E) \\ &\rightarrow E \times (E+E) \\ &\rightarrow I \times (E+E) \\ &\rightarrow b \times (E+E) \\ &\rightarrow b \times (I+E) \\ &\rightarrow b \times (b+E) \\ &\rightarrow b \times (b+I) \\ &\rightarrow b \times (b+I_1) \\ &\rightarrow b \times (b+I_{11}) \\ E &\rightarrow b \times (b+a_{11}) \end{aligned}$$

RED:

$$\begin{aligned} E &\rightarrow E \times E \\ &\rightarrow E \times (E) \\ &\rightarrow E \times (E+E) \\ &\rightarrow E \times (E+I) \\ &\rightarrow E \times (E+I_1) \\ &\rightarrow E \times (I+a) \\ &\rightarrow E \times (I+a_{11}) \\ &\rightarrow E \times (b+a_{11}) \\ &\rightarrow I \times (b+a_{11}) \\ &\rightarrow b \times (b+a_{11}) \end{aligned}$$

Pairs tree

$E \rightarrow E + E / E * E / (E) / a$

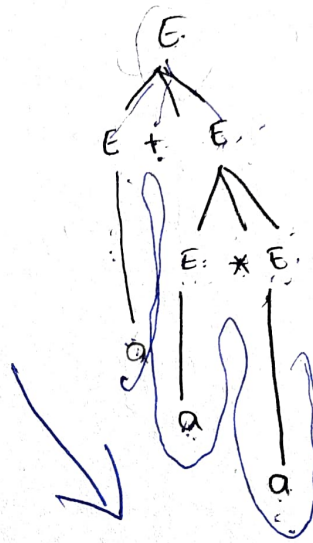
1) $a + a * a$

Soln:

LMD:

$E \rightarrow E + E$
 $\rightarrow E + E * E$
 $\rightarrow a + E * E$
 $\rightarrow a + a * E$
 $E \rightarrow a + a * a$

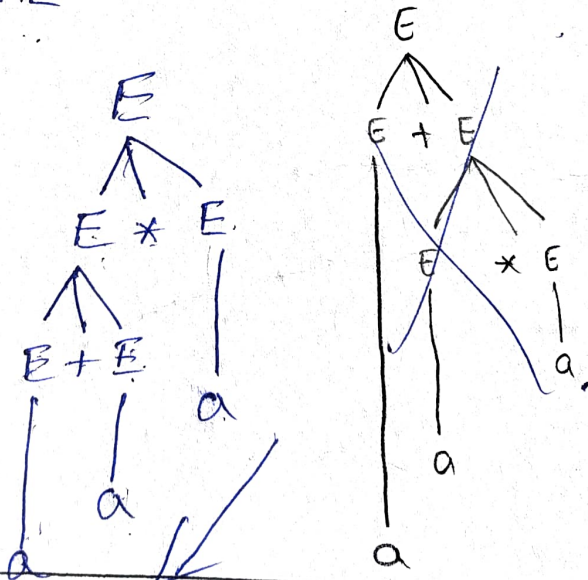
LMD for Parse tree



RMD:

$E \rightarrow E * E$
 $\rightarrow E + E * E$
 $\rightarrow E + E * a$
 $\rightarrow E + a * a$
 $\rightarrow a + a * a$

RMD for Parse tree



2) $(a + a) * a \Rightarrow E \rightarrow E + E / E * E / (E) / a$

LMD:

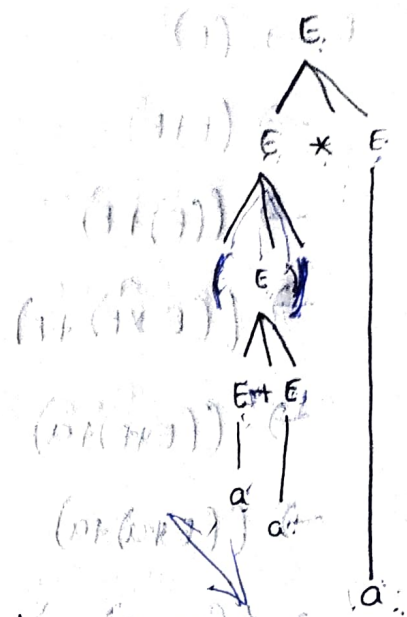
$E \Rightarrow E * E$
 $\Rightarrow (E) * E$
 $\Rightarrow (E + E) * E$

$$\Rightarrow (a + E) * E$$

$$\Rightarrow (a + a) * E$$

$$\Rightarrow (a + a) * a$$

Parse tree for LMD



RMD:

$$E \rightarrow E * E$$

$$\rightarrow (E) * E$$

$$\rightarrow (E + E) * E$$

$$\rightarrow (E + a) * a$$

$$\rightarrow (a + a) * a$$

$$\rightarrow (a + a) * a$$

Parse tree for RMD:



$$3) ((a * a) + a) \Rightarrow E \rightarrow E + E / E * E / (E) / a$$

LMD:

$$E \rightarrow (E)$$

$$\rightarrow (E + E)$$

$$\rightarrow ((E) + E)$$

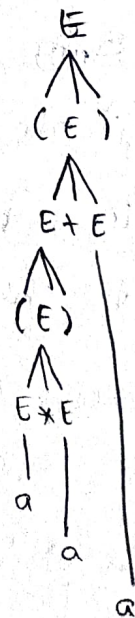
$$\rightarrow ((E * E) + E)$$

$$\rightarrow ((a * E) + E)$$

$$\rightarrow ((a * a) + E)$$

$$E \rightarrow ((a * a) + a)$$

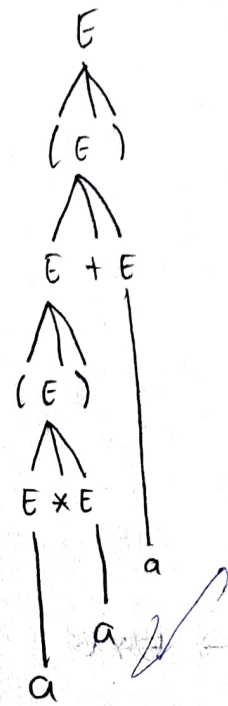
Parse tree for LMD



RMD:

$E \rightarrow (E)$
 $\rightarrow (E+E)$
 $\rightarrow ((E)+E)$
 $\rightarrow ((E * E) + E)$
 $\rightarrow ((E * E) + a)$
 $\rightarrow ((E * a) + a)$
 $\rightarrow ((a * a) + a)$

Parse tree for RMD



Push down Automata (PDA):

The Context free language is defined by a special type of automata namely Push down Automata. The Push down automata is an extension of non-deterministic finite automata with Epsilon (ϵ) transition with addition of the stack.

The stack is used to store the string of stack

Symbols i) The stack is used to read the symbols Push & Pop only at the top of the stack.

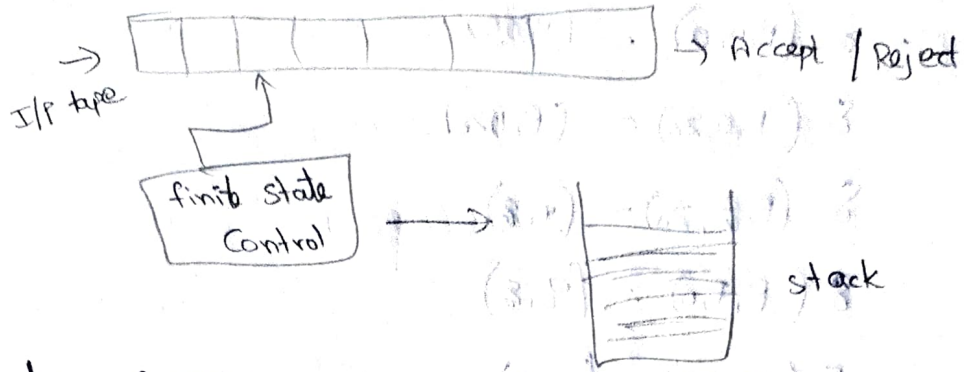
(ii) The push down automata can remember infinite amount of the string information.

iii) The Push down automata can access the information on its stack in last in first out.

iv) The Pushdown automata can recognize only the context free language (CFL)

The Pushdown automata is more powerful than finite automata which has finite memory.

PDA



Components of PDA:

The PDA consists of the following components

- i) finite state control (control unit)
- ii) I/P tape
- iii) Read head
- iv) stack

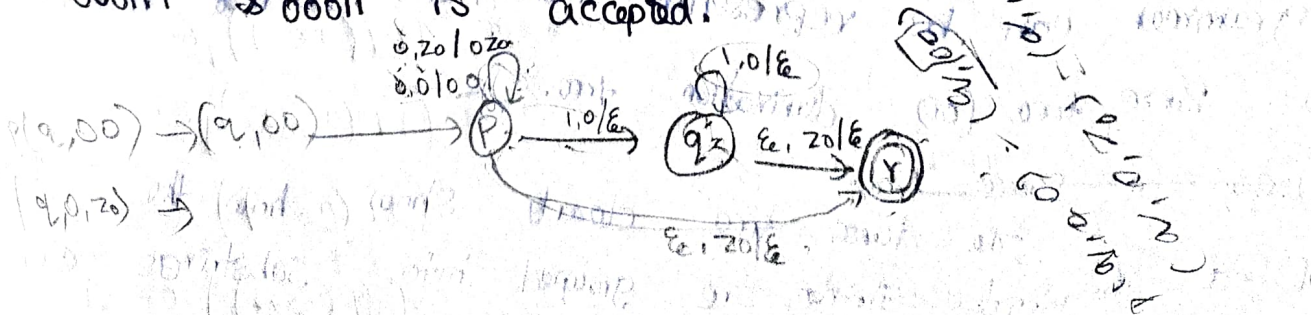
formal definition of PDA:

The PDA - or non-deterministic PDA (NPDA) involves of components.

$$Q, \Sigma, \Gamma, \delta, q_0, z_0, F$$

- where,
- Q - finite set of states
 - Σ - finite set of I/P symbols
 - Γ - finite state alphabet
 - δ - Transition function
 - q_0 - starting state of PDA
 - z_0 - starting symbol of stack (stack symbol)
 - F - final state of PDA (accepting state of PDA)

1) Consider the following PDA & find whether the string 000111 & 00011 is accepted.



$$\delta (P, 0, 0) = (P, 0, 0)$$

$$\delta (P, 0, z_0) = (P, 0, z_0)$$

$$\delta (P, \epsilon, z_0) = (r, \epsilon)$$

$$\delta (P, 1, 0) = (q, \epsilon)$$

$$\delta (q, 1, 0) = (q, \epsilon)$$

$$\delta (q, \epsilon, z_0) = (r, \epsilon)$$

00011

$$\delta (P, 00011, z_0)$$

$$= \delta (P, 0011, 0z_0)$$

$$= \delta (P, 011, 00z_0)$$

$$= \delta (P, 11, 000z_0)$$

$$= \delta (P, 1, 00z_0)$$

$$= \delta (P, \epsilon, 0z_0)$$

$$= \delta (P, \epsilon, z_0)$$

$$= (r, \epsilon)$$

$$00011 \quad \delta (P, 00011, z_0)$$

$$= \delta (P, 0011, 0z_0)$$

$$= \delta (P, 011, 00z_0)$$

$$= \delta (P, 11, 000z_0)$$

$$= \delta (P, 1, 00z_0)$$

$$= \delta (q, \epsilon, 0z_0)$$

$\delta (q, \epsilon, 0z_0)$ is not in the above transition function

So the string 00011 is not accepted.

Parse tree

The string that are derived from the context free grammar can be represented in a tree format is known as parse tree (or) derivation tree.

~~uses of parse tree:~~

The parse tree clearly show how the symbols of a terminal strings are grouped into substrings each one which belongs to the language of one of this variable of a grammar.

uses of parse tree:

The parse tree can be used for both derivation of recursive language.

The parse tree is used to find ambiguity in grammar & languages.

The parse tree is used in compiler which facilitate the translation the source program into executable code.

Ambiguity in Grammar & language:

If there is a grammar $G = (V, T, P, S)$ then the grammar G is ambiguous, if there is a string 'w' that can be derived two or more ways using some derivation method either LMD or RMD.

(or)

The grammar is ambiguous, if & only if there exist atleast one string 'w' for which two or more different parse tree exist by applying either LMD or RMD.

2

1) check whether the following grammar is ambiguous.

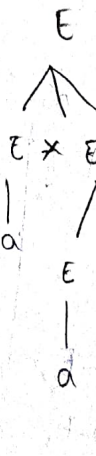
$$E \rightarrow E + E \mid E * E \mid E - E \mid a$$

Soln:

Consider the string $w = a * a + a$

first LMD:

$$\begin{aligned} E &\rightarrow E * E \\ E &\rightarrow E * E + E \\ E &\rightarrow a * E + E \\ &\rightarrow a * a + E \\ &\rightarrow a * a + a \end{aligned}$$



Second LMD:

$E \rightarrow E+E$
 $\rightarrow E \times E + E$
 $\rightarrow a \times E + E$
 $\rightarrow a \times a + E$
 $E \rightarrow a \times a + a$



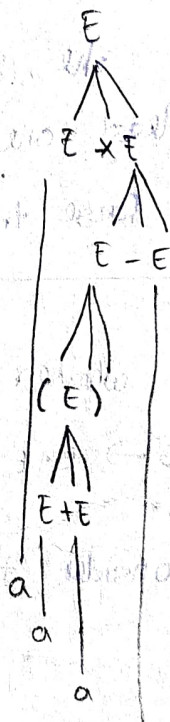
So, the string $a+a$ is derived from two LMD derived from the above grammar.

So, the grammar $E \rightarrow E+E / E \times E / E-E / a$ is ambiguous grammar.

2) $w = a \times (a+a) - a$ $E \rightarrow E+E / E \times E / E-E / (E) / a$

First LMD:

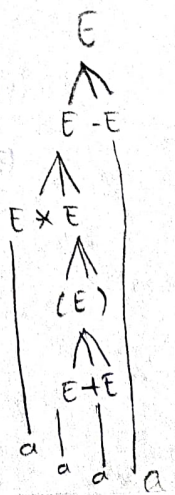
$E \rightarrow E \times E$
 $\rightarrow E \times E - E$
 $\rightarrow E \times (E) - E$
 $\rightarrow E \times (E+E) - E$
 $\rightarrow a \times (E+E) - E$
 $\rightarrow a \times (a+E) - E$
 $\rightarrow a \times (a+a) - E$
 $E \rightarrow a \times (a+a) - a$



Second LMD:

$E \rightarrow E - E$
 $\rightarrow E \times E - E$
 $E \times (E) - E$
 $E \times (E+E) - E$
 $a \times (E+E) - E$
 $a \times (a+E) - E$

$a \times (a+a) - E$
 $a \times (a+a) - a$



So the string $a^* (a+a) - a$ is derived from two left most derivation derived from the above grammar

So, the grammar $E \rightarrow E+E / E * E / E - E / (\epsilon) / a$ is ambiguous grammar.

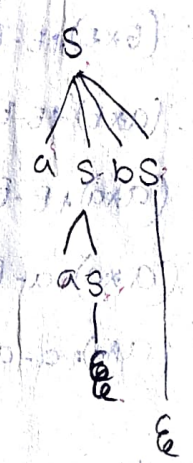
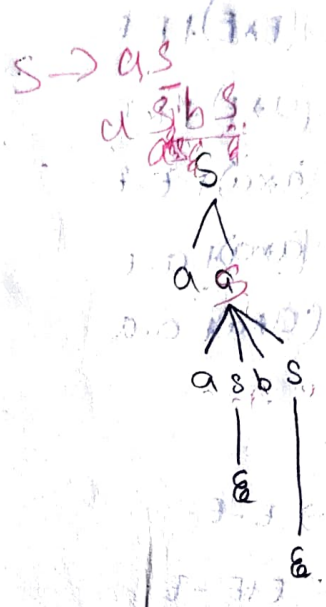
3) $S = as / asbs / \epsilon$ 'aab'

First LMD

$S \rightarrow as$
 $\rightarrow aasbs$
 $\rightarrow aa \epsilon bs$
 $\rightarrow aa \cdot bs$
 $\rightarrow aab \epsilon$
 $S \rightarrow aab$

Second LMD:

$S \rightarrow asbs$
 $\rightarrow aa \cdot sbs$
 $\rightarrow aa \epsilon bs$
 $\rightarrow aabs$
 $\rightarrow aab \epsilon$
 $S \rightarrow aab$



So the string 'aab' is derived from two LMD derived from the above grammar.

So, the grammar $S \rightarrow as / bsas / \epsilon$ is ambiguous grammar.

Removing Ambiguity from the Grammar of unambiguity:

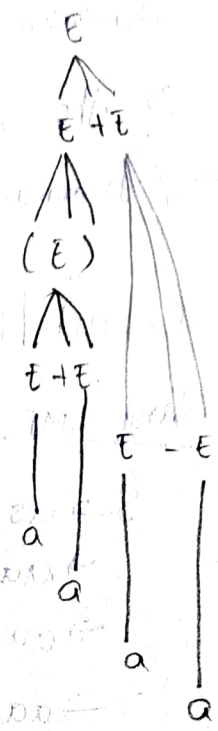
1. Check whether the following CFG is ambiguous and remove the ambiguous & make the grammar has unambiguous.

$E \rightarrow E * E / E + E / E - E / (\epsilon) / a$

first CMD: let us consider the string $(axa) + a - a$

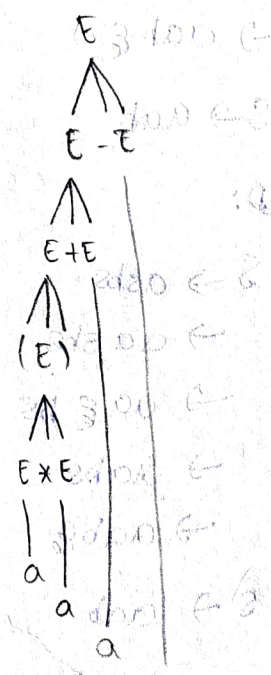
$a + (axa) = a$
 $a - (a + b) = a$

- $E \rightarrow E + E$
- $\rightarrow (E) + E$
- $\rightarrow (E * E) + E$
- $\rightarrow (E * E) + E - E$
- $\rightarrow (a * E) + E - E$
- $\rightarrow (a * a) + E - E$
- $\rightarrow (a * a) + a - E$
- $\rightarrow (a * a) + a - a$



Second CMD:

- $E \rightarrow E - E$
- $\rightarrow E + E - E$
- $\rightarrow (E) + E - E$
- $\rightarrow (E * E) + E - E$
- $\rightarrow (a * E) + E - E$
- $\rightarrow (a * a) + E - E$
- $\rightarrow (a * a) + a - E$
- $\rightarrow (a * a) + a - a$



we have a two parse tree for a single string $(axa) + a - a$.
 So, the above grammar is ambiguous grammar.

Unambiguous:

$$E \rightarrow E * E \mid E + E \mid E - E \mid E / E \mid (E) \mid a$$

$$E \rightarrow E * T \mid E + T$$

$$T \rightarrow E - X \mid E / X$$

$$X \rightarrow (E) \mid a$$

first CMD:

$(a * a) + a - a$

$$(axa) + a - a$$

$$E \rightarrow E + T$$

$$\rightarrow E + E - X$$

$$E \rightarrow E + E - a$$

So, the string $(axa) + a - a$ is not derived from the above grammar

So, the above grammar is unambiguous.

$$2) E \rightarrow I / E * E / E \wedge E / (E)$$

$$I \rightarrow I_a / I_b / I_0 / I_1 / a/b$$

let us consider the string $ax \cdot b + b00$

first UMD:

$$E \rightarrow E * E$$

$$\rightarrow E \wedge E + E$$

$$\rightarrow I \wedge E + E$$

$$\rightarrow a \wedge E + E$$

$$\rightarrow a \wedge I + E$$

$$\rightarrow a \wedge b + E$$

$$\rightarrow a \wedge b + I$$

$$\rightarrow a \wedge b + I_0$$

$$\rightarrow a \wedge b + I_{00}$$

$$\rightarrow a \wedge b + b00$$

second UMD:

$$E \rightarrow E + E$$

$$\rightarrow E * E + E$$

$$\rightarrow I \wedge E + E$$

$$\rightarrow a \wedge E + E$$

$$\rightarrow a \wedge I + E$$

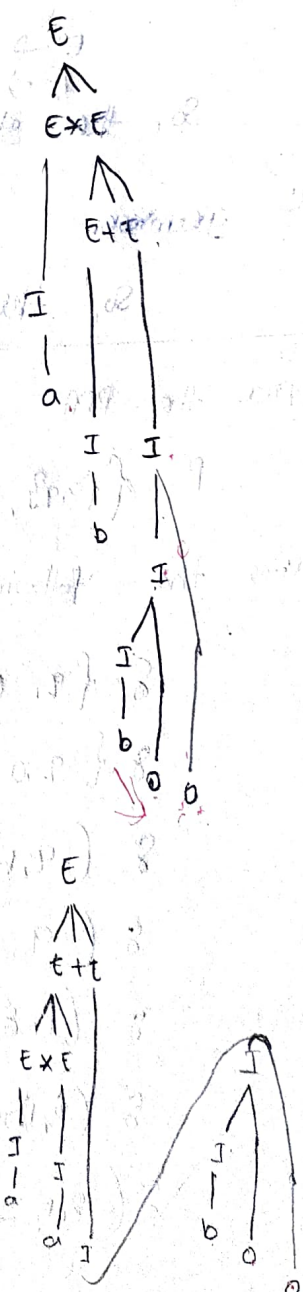
$$\rightarrow a \wedge b + E$$

$$\rightarrow a \wedge b + I$$

$$\rightarrow a \wedge b + I_0$$

$$\rightarrow a \wedge b + I_{00}$$

$E \rightarrow E * E$
 $\rightarrow I \wedge E$
 $\rightarrow a \wedge E$
 $\rightarrow a \wedge I$
 $\rightarrow a \wedge I_b$
 $\rightarrow a \wedge I_{bb}$
 $\rightarrow a \wedge a_{bb}$



So, the string $a^*b^*b^*$ is derived from two LMD.

So, the above grammar is ambiguous grammar.

Unambiguous:

$$E \rightarrow \underline{I} / \underline{EXE} / \underline{E^*E} / \underline{(E)}$$

$$I \rightarrow \underline{I_0} / \underline{I_1} / \underline{a/b}$$

$$E \rightarrow \underline{X} / \underline{EXE}$$

$$X \rightarrow E^* / T^* / (E)$$

$$T \rightarrow z_0 / z_1$$

$$z \rightarrow z_0 / z_1 / a/b$$

first LMD: $a^*b^*b^*$

$$E \rightarrow EXX$$

$$E \rightarrow X$$

So, the string $a^*b^*b^*$ is not derived from the above grammar.

So, the above grammar is unambiguous.

1) Suppose the PDA

$$P = \{p, q, r, s, z_0, x\}, \delta, q_0, z_0, \{p, q\}$$

assume the following transition function.

$$\delta(q, 0, z_0) = \{q, xz_0\}$$

$$\delta(q, 0, x) = \{q, xx\}$$

$$\delta(q, 1, x) = \{q, x\}$$

$$\delta(q, \epsilon, x) = \{p, \epsilon\}$$

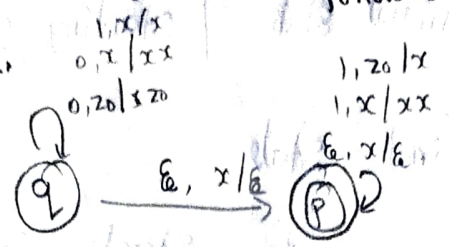
$$\delta(p, \epsilon, x) = \{p, \epsilon\}$$

$$\delta(p, 1, x) = \{p, xx\}$$

$$\delta(p, 1, z_0) = \{p, x\}$$

when the S/P string w as follows $w = 0011$, $w = 010$ is accepted or not.

Soln:



$w = 0011$

$$\delta(q, 0011, z_0) \rightarrow \delta(q, 011, xz_0)$$

$$\Rightarrow \delta(q, 011, 0z_0) \rightarrow \delta(q, 11, xx)$$

$$= \delta(q, 11, 00z_0) \rightarrow \delta(p, 1, x)$$

$$= \delta(p, 1, 0z_0) \rightarrow \delta(p, \epsilon, xx)$$

$$= \delta(p, \epsilon, z_0) \rightarrow \delta(p, \epsilon)$$

$\therefore p$ is a final state $w = 0011$ is accepted.

$w = 010$

$$\delta(q, 010, z_0) \rightarrow \delta(q, 010, z_0)$$

$$\delta(q, 10, 0z_0) \rightarrow \delta(q, 10, xz_0)$$

$$\delta(p, 0, z_0) \rightarrow \delta(p, 0, x)$$

$(p, 0, z_0)$ is not in the transition function.

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design PDA to accept the language $L = \{a^n b^n / n \geq 1\}$ accepting by final state.

$$(q, 0, p) = (q, 0, p)$$

design PDA for the set of all string $\{a, b\}$ with equal

no. of a's & equal no. of b's

Soln:
Here, the string contains equal no of a's & b's it also have the empty string.

let $q_0 \rightarrow$ initial state / starting state

$q_2 \rightarrow$ final state

$z_0 \rightarrow$ top of the stack symbol.

The idea to design this PDA is that when ever we read 'a' initially push all a's to the stack.

we read all b's we delete 'a' from the stack. that is for each 'a' in the stack we delete 'a' from the stack for read each 'b'

When we reach the empty string, then the string contains z_0 symbols.

after reading the entire string, we delete the stack symbols z_0 & reach the final state.

$$L = \{a^n b^n \mid n \geq 1\}$$

@abbcc

$$(q_0, a, z_0) = (q_0, a z_0)$$

$$(q_0, a, a z_0) = (q_0, a a z_0)$$

$$(q_0, b, a a z_0) = (q_1, a z_0)$$

$$(q_1, b, a z_0) = (q_1, z_0)$$

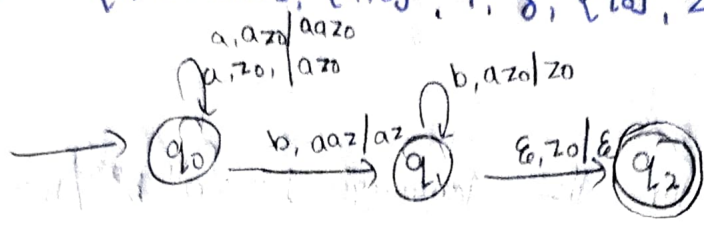
$$(q_1, \epsilon, z_0) = (q_2, \epsilon)$$

$a_1 z_0 / a z_0$
 $a_1 a z_0 / a a z_0$

$b_1 a a z_0 / a z_0$
 $b_1 a z_0 / z_0$

PDA:

$$P = \{ \{q_0, q_1, q_2\}, \{a, b\}, \gamma, \delta, \{q_0\}, z_0, \{q_2\} \}$$



Consider the s/p string 'aabb' & 'aab'

aabb

$$\begin{aligned} & (q_0, aabb, z_0) \\ &= (q_0, abb, aaz_0) \\ &= (q_0, bb, aaz_0) \\ &= (q_1, b, aaz_0) \\ &= (q_1, \epsilon, z_0) \\ &\rightarrow (q_2, \epsilon) \end{aligned}$$

q_2 is final state. So 'aabb' is accepted.

'aab'

$$\begin{aligned} & (q_0, aab, z_0) \\ &= (q_0, ab, aaz_0) \\ &= (q_0, a, aaz_0) \\ &= (q_1, \epsilon, aaz_0) \end{aligned}$$

(q_1, ϵ, aaz_0) is not in the PDA of above transition function.
 ~~q_1 is not final state, so 'aab' is not~~
 So, 'aab' is not

accepted.

2) design PDA to accepted the language $L =$

$$\{0^n 1^n / n \geq 1\}$$

3) design PDA for the language $L = \{a^n b^{2n} / n \geq 1\}$ is accepting by final state
(or)

design PDA with the set of string with twice many b's than a's with as the starting string.

Soln:

the idea to design PDA is that when we read single 'a' we push two a's on the state

When we read 'b' we pop each 'a' top of the stack Z_0 , we reach the final state

let $q_0 \rightarrow$ starting state
 $q_2 \rightarrow$ final state
 $Z_0 \rightarrow$ top of the stack symbol.

$$L = \{a^n b^{2n} / n \geq 1\}$$

aaabbbb

$$(q_0, a, Z_0) = (q_0, aaZ_0)$$

$$(q_0, a, aaZ_0) = (q_0, aaaaZ_0)$$

$$(q_0, b, aaaa z_0) = (q_1, aaaa z_0)$$

$$(q_1, b, aaaa z_0) = (q_1, aaaa z_0)$$

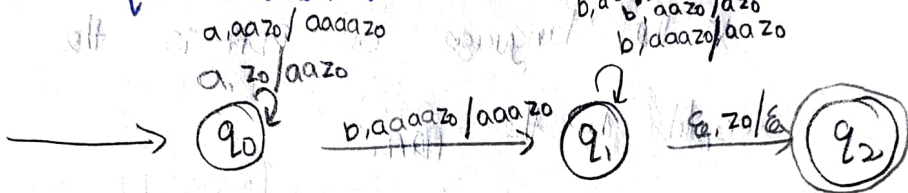
$$(q_1, b, aaaa z_0) = (q_1, aaaa z_0)$$

$$(q_1, b, aaaa z_0) = (q_1, aaaa z_0)$$

$$(q_1, \epsilon, z_0) = (q_2, \epsilon)$$

PDA:

$$P = \{ \{q_0, q_1, q_2\}, \{a, b\}, \gamma, \delta, \{q_0\}, z_0, \{q_2\} \}$$



Consider the IP string aabb & aab

$$(q_0, aabb, z_0)$$

$$= (q_0, abb, az_0)$$

$$= (q_0, bb, aaz_0)$$

$$= (q_1, b, aaz_0)$$

$$= (q_1, \epsilon, az_0)$$

$$= (q_2, \epsilon)$$

q_2 is final state & 'aabb' is accepted.

let 'aab'

$$\rightarrow (q_0, aab, z_0)$$

$$\rightarrow (q_0, ab, az_0)$$

(q_0, b, aaz_0)

(q_1, ϵ, az_0)

$\therefore (q_1, \epsilon, az_0)$ is not in the PDA of above transition function. So 'aab' is not accepted.

18-8-15

language to PDA / language of PDA:

* The language of PDA is the set of all strings accepted by the PDA.

* Generally, the string is accepted by PDA after consuming the input, the PDA should either final state.

* Else after reading the I/P, stack should be empty, then the language is accepted by PDA.

* So, two methods on accepting a string in the PDA as follows.

- 1) acceptance by final state
- 2) acceptance by empty stack

1. Acceptance by final state

let $P = \{Q, \epsilon, \gamma, \delta, \{q_0\}, z_0, F\}$

be a PDA then the language $L(P)$ is accepted by

final state

$$L(P) = \{w/q_0, w, z_0, T_p, (q, \epsilon, \alpha)\}$$

q - final state

α - any stack symbol

acceptance by empty ~~stack~~ stack

$$\text{let } P = \{q, \epsilon, \gamma, \delta, \{q_0\}, z_0, F\}$$

be a PDA then the language $L(P)$ is accepted by the empty stack.

$$L(P) = \{w/q_0, w, z_0, T_p, (q, \epsilon, \alpha)\}$$

Problem on converting (CFG to) PDA:

1) ^{13m} Construct PDA for the following grammar:
 $E \rightarrow E+E \mid E * E \mid a$

Soln:

non terminal $\Rightarrow E$

$$\delta(q, E) = \{(q, E+E), (q, E * E), (q, a)\}$$

Terminal $\Rightarrow +, *, a$

$$\delta(q, +, +) = (q, \epsilon)$$

$$\delta(q, *, *) = (q, \epsilon)$$

$$\delta(q, a, a) = (q, \epsilon)$$

Consider the I/P string $w = 'a * a + a'$

transition function:

$$\rightarrow \delta(q, a \times a + a, \epsilon)$$

$$\rightarrow \delta(q, a \times a + a, \epsilon \times \epsilon)$$

$$\rightarrow \delta(q, a \times a + a, a \times \epsilon)$$

$$\rightarrow \delta(q, \times a + a, \times \epsilon)$$

$$\rightarrow \delta(q, a + a, \epsilon)$$

$$\rightarrow \delta(q, a + a, \epsilon + \epsilon)$$

$$\rightarrow \delta(q, a + a, a + \epsilon)$$

$$\rightarrow \delta(q, + a, + \epsilon)$$

$$\rightarrow \delta(q, a, \epsilon)$$

$$\rightarrow \delta(q, a, a)$$

$$\rightarrow \delta(q, \epsilon, \epsilon)$$

Thus the CFG accept the string 'a * a + a' and it's accepted by PDA in empty stack.

Construct PDA for following grammar

$$E \rightarrow E * E \mid E + E \mid E - E \mid E / E \mid \epsilon$$

$$I \rightarrow I_a \mid I_b \mid I_1 \mid I_0 \mid a/b$$

Terminal $\rightarrow a, \dots, z, +, -, *, /$
also ϵ

non-terminal $\rightarrow A, \dots, Z$

Soln:

non terminal: E, I

$$\delta(q, \epsilon) = \{(q, \epsilon * \epsilon) (q, \epsilon + \epsilon) (q, \epsilon - \epsilon) (q, \epsilon / \epsilon), (q, \epsilon)\}$$

$$\delta(q, \Gamma) = \{ (q, \Gamma a), (q, \Gamma b), (q, \Gamma \epsilon), (q, \Gamma \cdot), (q, a), (q, b) \}$$

terminal: *, +, -, /, a, b, 0, 1, ...

transition function:

$$\delta(q, *, *) = (q, \epsilon)$$

$$\delta(q, +, +) = (q, \epsilon)$$

$$\delta(q, -, -) = (q, \epsilon)$$

$$\delta(q, /, /) = (q, \epsilon)$$

$$\delta(q, a, a) = (q, \epsilon)$$

$$\delta(q, b, b) = (q, \epsilon)$$

$$\delta(q, 0, 0) = (q, \epsilon)$$

$$\delta(q, 1, 1) = (q, \epsilon)$$

Consider the I/P string 'axatbll'

transition function:

$$= \delta(q, axatbll, \epsilon)$$

$$= \delta(q, axatbll, \epsilon * \epsilon)$$

$$\delta(q, axatbll, \Gamma * \epsilon)$$

$$\delta(q, axatbll, a * \epsilon)$$

$$\delta(q, *atbll, * \epsilon)$$

$$\delta(q, atbll, \epsilon)$$

$$\delta(q, atbll, \epsilon * \epsilon)$$

$$\delta(q, a+b11, \mathbb{I}+\epsilon)$$

$$\delta(q, a+b11, a+\epsilon)$$

$$\delta(q, +b11, +\epsilon)$$

$$\delta(q, b11, \epsilon)$$

$$\delta(q, b11, \mathbb{I})$$

$$\delta(q, b11, \mathbb{I}_1)$$

$$\delta(q, b11, \mathbb{I}_{11})$$

$$\delta(q, b11, b11)$$

$$\delta(q, 11, 11)$$

$$\delta(q, 1, 1)$$

$$\delta(q, \epsilon, \epsilon) \text{ (or) } \delta(q, \epsilon)$$

accepted by PDA Thus CFG accept the string 'a* a+b11' & in empty stack.

$$S \rightarrow aAA$$

$$A \rightarrow as/b1/a$$

$$3) S \rightarrow AA / 0$$

$$A \rightarrow 00/1$$